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Magnetohydrodynamic Vortex Containment, Part 4: System Performance Assessment

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This is the last in a series of papers on the effectiveness of using magnetohydrodynamically driven vortices to contain uranium in a gas core nuclear propulsion system. This paper evaluates the overall system performance, with particular focus on optimizing the power conversion system that supplies electrical power to the magnetohydrodynamic drive. The system is unique in that it is essentially nuclear thermal, however, it requires a large recirculation of electrical power. Although the assumed power conversion system is thermodynamic (Brayton) it is shown that this conversion can be accomplished either with a greatly reduced radiator area or, under some circumstances, without the need for radiators at all, by sinking waste heat into the propellant stream. A baseline set of system parameters is presented.

Nomenclature

A_{rad}	=	radiator area
c_p	=	specific heat capacity
F	=	thrust
f	=	solid phase fission fraction
g	=	gravitational acceleration
I_{sp}	=	specific impulse
J_p	=	vortex perimeter current density
$\dot{m}$	=	mass flow rate
η	=	cycle efficiency
η_c	=	Carnot cycle efficiency
$\dot{q}$	=	heat transfer rate
MHD	=	magnetohydrodynamic parameter
P	=	power
P_{fission}	=	fission power
P_{prop}	=	propellant feed power
P_{rad}	=	radiated power
P_s	=	solid phase power generation
P_{thrust}	=	thrust power
P_v	=	vortex cavity power generation
Q	=	heat
r_m	=	radius of peak uranium density
T	=	temperature
W	=	engine weight
w	=	work
β	=	power generation parameter
β_v	=	reactor void fraction
σ	=	Stefan–Boltzmann constant

Introduction

TO achieve the specific power necessary for exploration of the solar system in reasonable amounts of time, it is necessary to use fuels that themselves provide a much higher power density than can be achieved by chemical propulsion. Nuclear fission combined with a light propellant, such as hydrogen, provides both the power density and increased specific impulse required to achieve these shorter trip times. However, the full capability of these fuels to provide high specific impulse is not currently recognizable in a conventional nuclear thermal rocket, because the temperatures that can be achieved are limited by the melting temperatures of the solid fuel rods. The use of fuels in the gas phase provides a way of bypassing this immediate issue; however, it poses other unique challenges. For instance, how to confine the fuel until it is consumed by the reaction, while effectively transferring the reaction energy to the propellant and isolating the reactor assembly from the high propellant temperatures that result.

As discussed in the first paper of this series [1], one early nuclear fuel containment scheme developed by Kerrebrock and Meghreblian [2] considered the use of a hydrodynamic vortex to establish a pressure gradient within a cylindrical cavity. This gradient could push propellant through a heavy fissile fuel, while at the same time keeping it isolated from the walls of the cavity. Problems with this approach caused it to be abandoned; however, it was conjectured that by introducing magnetohydrodynamic forces into the vortex, a workable system could be achieved. Thus far, details of the chemical and transport properties have been presented [3], as well as an axisymmetric flow analysis that demonstrated the theoretical limit on specific impulse [4]. To complete the analysis, the details of the power system design and how it can be optimized are presented, demonstrating that the waste heat can actually be recycled into the propellant flow, eliminating the need for radiators.

Much of the analysis presented by Kerrebrock and Meghreblian [2] focused on containment and dealt with how the flow solutions varied with values of w_m, M_{tm}, and $g_m/g_{m(\max)}$. Once these dependencies were established, the performance level of the system as a whole was considered. The analysis progressed by considering how power was transferred through the system, from its generation during the fission process to its final conversion to thrust power. A similar approach will be taken here, beginning with the original results for comparison. A baseline set of parameters is then chosen, and the overall performance of the system is presented.

Power System

The useful energy released as the result of the fission of enriched uranium is roughly 195 MeV, the initial form of which is kinetic and is divided among the fission fragments, neutrons, neutrinos, betas, and gammas. The fission fragments and the beta particles (electrons) are electrically charged and, as a result, deposit their energy as heat very close to the fission site, over the course of many Coulomb-type collisions. High-energy photons (gammas) that are generated also interact with charged particles in the reactor by scattering or are themselves converted to charged particles through pair production. In either case, the gammas may travel centimeters or even meters

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Table 1 Energy partition of fission products

Particle	Energy [MeV]	Deposited
Nuclei	168	Near
Neutrons	5	Far
Gammas	14	Far
Betas	8	Near
Neutrinos	12	Externally

before most of their energy is deposited. This is also true for neutrons, which are not affected by the Coulomb force, and even more so for the neutrinos, because of their negligibly small collision probability. Each particle type carries a fraction of the total energy with it, and the distribution of the fission energy among the various products is given in Table 1 [5].

The total energy listed in this table is 207 MeV, however, energy from the neutrinos cannot be contained and should not be considered as available to the reactor. Because of the much higher particle density of the moderator of the reactor, it can be assumed that most of the far-reaching neutron and gamma energy will eventually be deposited in this region. From the table, this means that of the total power generated in the gas phase, approximately 10% of it will escape the confines of the vortex tubes and appear in the solid regions of the reactor. These regions must therefore be cooled and, for a purely hydrodynamic system, the most obvious solution is regenerative cooling by passing the low-temperature propellant through the moderator regions before injection into the vortex chambers.

For many reasons, it is also desirable to have some fraction of the total fission power generated in the solid regions of the system, thus allowing the reactor to be operational and generating power in the absence of propellant flow. Two such instances when this could be favorable are during startup, where gas phase containment cannot be maintained, and during idle, when power is needed without propulsion. The total heat deposited in the solid regions is then the sum of what is generated there directly by fission and what is deposited there by radiation from fission in the gas phase. This is given quantitatively by

$$P_s = fP + 0.1P_v = [f + 0.1(1 - f)]P \tag{1}$$

where $P_v = (1 - f)P$ is the fission power generated in the vortex cavity, and the 0.1 is a consequence of the 10% that is transported away. If the heat deposited in the solid region was the sole source of power in the system ($f = 1$), it would operate as a solid core rocket at a power level of P_s when propellant was introduced. In a gas core concept, additional heat is added to the propellant in the gas phase, until the total power has been supplied. The thrust power realized can be expressed in terms of the specific impulse of the engine as

$$P_{\text{thrust}} = \tfrac{1}{2}\dot{m}(gI_{\text{sp}})^2 \tag{2}$$

Applying this relationship to both concepts (under the assumption that all power goes to thrust), the ratio of specific impulse achievable by the gas core system (variable f) to that of the solid core ($f = 1$) becomes

$$I_{\text{sp}_v}/I_{\text{sp}_s} = [f + 0.1(1 - f)]^{-1/2} \tag{3}$$

In the extreme case, where all of the fission takes place in the gas phase ($f = 0$), this would mean an I_{sp} increase of roughly a factor of 3 above a solid core system, achieving nearly 2500 s. This is the maximum performance increase that can be obtained using this system. The limiting factor is the need to cool the solid regions against the 10% of the total heat generated that ends up there, using only the available propellant flow. To further increase the specific impulse, some additional means of extracting heat from the moderator region must be employed. A possible solution would be to introduce a secondary flow network to the moderator region to transport heat to external radiators. The ratio of heat added in the gas phase to that in the solid phase would then no longer be fixed by the requirement that all heat deposited in the solid region be extracted by the propellant.

For MHD-driven systems, it is necessary to generate electric power. The natural choice for this power source is from the heat generated or deposited in the solid regions of the reactor and is extracted through the employment of heat engines. The implementation of this heat cycle is similar to the secondary flow idea mentioned previously, but rather than simply radiating away the excess heat, a significant portion is converted to electrical energy. A necessary consequence of the second law of thermodynamics is that not all of the energy can be converted to electricity and that some waste heat must still be generated. However, as the efficiency of the power conversion cycle increases, the amount of waste heat will decrease. Introduction of this power system then has the threefold advantage of 1) providing power for the MHD system (as well as other spacecraft systems), 2) decoupling the specific impulse from the heat deposited in the solid regions (allowing higher I_{sp}), and 3) reducing the amount of waste heat that must be removed from the system. Because presently the only method of heat rejection is through the use of radiators, reduction of waste heat can lead to substantial weight savings. Performance of the system must be determined through an analysis of its power flow.

Heat engines operate more efficiently at lower rejection temperatures, whereas radiators function better at higher temperatures. The first statement is readily verified from the equation of efficiency for a Carnot cycle operating between high and low temperatures T_H and T_L respectively, where the relationship is

$$\eta_c = 1 - \frac{T_L}{T_H} \tag{4}$$

Because the power given off by radiators is governed by the equation of black body radiation, it is clear that not only does the effectiveness of radiators depend on temperature, it is highly sensitive to it ($q_r = \sigma T^4$). An analysis of the trade-off using a Carnot cycle shows that for a given heat source temperature (T_H), the radiator temperature that minimizes radiator area (and therefore radiator mass) is $3T_H/4$. For the current concept, a well-developed heat engine possessing the capacity to generate a large amount of power should be employed and, for this reason, the Brayton cycle [6] (found in most turbomachinery) will be considered. The ideal cycle is shown on a temperature–entropy diagram in Fig. 1.

Legs 1–2 and 3–4 represent isentropic compression and expansion, respectively, and legs 2–3 and 4–1 are heat addition and removal at constant pressure. Because the heat transfer occurs at constant pressure, it can be shown that the efficiency of the Brayton cycle is the same as that of a Carnot cycle operating between temperatures T_2 and T_1 or T_3 and T_4.

A schematic of how the cycle would be implemented is given in Fig. 2. The working fluid passes through the moderator region of the reactor, where it is heated to the maximum material temperature. The fluid is expanded through a turbine, which extracts power from the flow and uses it to both generate electricity and run the compressor. Upon exiting the turbine, the fluid is then passed through an external

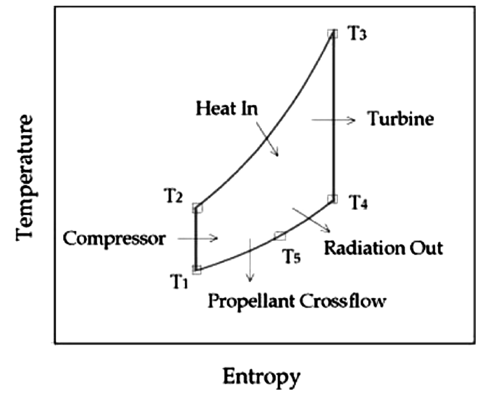

Fig. 1 Brayton cycle.

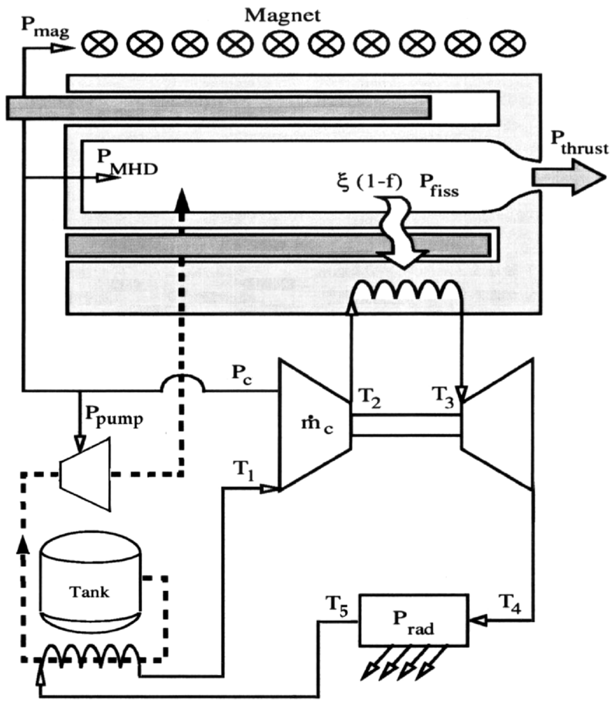

Fig. 2 Schematic of power cycle.

radiator, where power is removed by radiating it to free space. Additional cooling is then accomplished by crossflow heat exchange with the propellant coming from the tank. Finally, the low-temperature fluid is compressed before returning to the reactor core. After the heat exchange with the working fluid of the power cycle, the propellant passes through the moderator section as well, to be heated to the vortex injection temperature.

Ideally, all the waste heat from the power cycle would be removed by transferring it to the propellant before its entering the moderator region, thus eliminating the need for any radiators. Whether this is feasible will depend on the heat capacity of the propellant, as well as on its flow rate relative to that of the working fluid of the power cycle. Even if the radiators cannot be completely eliminated, significant reduction of their mass is still possible.

When considering only radiators for heat rejection, the optimization is done while holding the work per cycle constant. All heat transfer may be considered as per unit mass of the (circulating) working fluid, with the flow rate then determining the power. However, in the case where some heat is transferred to a second fluid (the propellant), the relative flow rates come into play, and it is the power that must be held constant. Relating the work generated per unit mass to the heat transferred from the high-temperature reservoir by $w = \eta Q_H$, this condition is given by

$$P_c = \dot{m}_c w = \dot{m}_c \eta Q_H = \dot{m}_c \eta c_{p_c}(T_3 - T_2) = \text{const} \tag{5}$$

Defining $x = T_2/T_3$, $\bar{m} = c_{p_p}\dot{m}_p/c_{p_c}\dot{m}_c$ and $\beta = P_c/\dot{m}_p c_{p_p} T_3$ (where the subscript p refers to the propellant and c to the cycle), the condition of constant power can then be written as

$$\beta = \frac{\eta(1-x)}{\bar{m}} = \text{const} \tag{6}$$

To relate the area (and hence mass) of the radiator to the temperature change across it, a balance must be achieved between the rate at which heat is removed from the propellant and the rate at which it is radiated away. The rate of heat transfer from a mass m is related to the rate of change in temperature through the expression

$$\mathrm{d}\,\dot{q} = c_p m \frac{\mathrm{d}T}{\mathrm{d}t} \tag{7}$$

or in the frame of reference where the mass is flowing by at a certain flow rate $\dot{m}$

$$\mathrm{d}\,\dot{q} = c_{p_c}\dot{m}_c\,\mathrm{d}T \tag{8}$$

The heat transfer rate through a differential element of radiator area is given from the radiation law as

$$d\dot{q} = dA_{\text{rad}}\sigma T^4 \tag{9}$$

where an emissivity of one has been arbitrarily chosen.

Equating the two heat transfer rates, the differential equation that results is

$$\dot{m}_c c_{p_c}\,\mathrm{d}T = \mathrm{d}A\sigma T^4 \tag{10}$$

which may be integrated across the radiator between T_4 and T_5 to achieve the result

$$A = \frac{\dot{m}_c c_{p_c}}{3\sigma}\left[\frac{1}{T_5^3} - \frac{1}{T_4^3}\right] \tag{11}$$

To get this expression in a dimensionless form, it was found convenient to define an area A_m, given by the expression

$$A_m = \frac{P_c}{\sigma T_3^4} \tag{12}$$

Factoring $1/T_3^3$ out of the expression for area and nondimensionalizing by A_m, the resulting equation is then

$$\bar{A} = \frac{A}{A_m} = \frac{1}{3\beta\bar{m}}\left[\left(\frac{T_3}{T_5}\right)^3 - \frac{1}{(1-\eta)^3}\right] \tag{13}$$

where the ratio T_4/T_3 has been written in terms of the cycle efficiency. The ratio of T_5 to T_3 can be found by relating the waste heat to the heat drawn from the high-temperature reservoir in the same way as it was done for the work. Because an amount η of the available energy becomes work, the amount rejected is then $(1 - \eta)$. The waste heat balance can then be written as

$$\dot{m}_c c_{p_c}(T_3 - T_2)(1-\eta) = \dot{m}_c c_{p_c}(T_4 - T_5) + \dot{m}_p c_{p_p}(T_m - T_T) \tag{14}$$

The term on the left is the fraction of heat supplied that is not used as work. On the right, the first term is the amount of heat that is radiated into space, and the second term is the amount of heat that is transferred to the propellant as it flows from the tank. This has been written in terms of the propellant flow rate, with the low temperature that of the tank, and the maximum temperature labeled as T_m. For different relative mass flow rates between the propellant and the heat engine working fluid, it is unlikely that the maximum temperature achieved by the propellant during the crossflow heat exchange will equal that of the exit temperature of the radiator. However, the propellant can certainly not exceed this temperature, as heat would then be flowing in the other direction, and if it falls short of this temperature, the system does not take full advantage of the propellant's heat capacity. Therefore, the condition that the maximum temperature reached by the propellant equal that of the exit temperature of the radiator ($T_m = T_5$) is used to define the relative mass flow rate that will maximize heat transfer to the propellant. Again dividing the temperatures by T_3 and making the substitutions defined earlier, this balance can be written

$$(1-x)(1-\eta) = (1-\eta) - \bar{m}\frac{T_T}{T_3} + (\bar{m}-1)\frac{T_5}{T_3} \tag{15}$$

Each term is roughly of order one, except for the second term on the right, which can be neglected relative to the others, as using even noncryogenic propellant would set the ratio T_T/T_3 to about a tenth. Finally, this can be solved for the ratio of T_5/T_3 to yield

$$\frac{T_5}{T_3} = \frac{x(1-\eta)}{1-\bar{m}} \tag{16}$$

and then substituted into Eq. (13) to find

$$\bar{A} = \frac{1}{3(1-x)\eta(1-\eta)^3}\left[\left\{1 - \frac{\eta(1-x)}{\beta}\right\}^3 x^{-3} - 1\right] \tag{17}$$

where $\bar{m}$ has been eliminated using Eq. (6). Looking at the term in square brackets, the area of the radiator will go to zero under the condition

$$\left\{1 - \frac{\eta(1-x)}{\beta}\right\}^3 = x^3 \tag{18}$$

which may be simplified to the result $\eta = \beta$. From Eq. (6), this also implies that $\bar{m} = 1 - x$. The implication of this result is that if the power required from the heat engine is low enough that β (the ratio of P_c to $\dot{m}_p c_{p_p} T_3$) is less than one, matching the efficiency of the heat engine to this value will eliminate the need for radiators. A quick check of the feasibility of this condition during operation can be made by examining the following relationship [4]:

$$\frac{P_{\text{mhd}}}{P_{\text{thrust}}} = 6\alpha(1 + \text{MHD})\text{MHD}\left(\frac{v_m}{I_{\text{sp}} g}\right)^2 \tag{19}$$

The parameter β can be written as

$$\beta = \frac{P_c}{\dot{m}_p c_{p_p} T_3} = \left(\frac{P_c}{P_{\text{thrust}}}\right)\left(\frac{P_{\text{thrust}}}{\dot{m}_p c_{p_p} T_3}\right) \tag{20}$$

and assuming that the power demands of other components of the spacecraft are not significant as compared with the MHD system, the first ratio is just that defined in Eq. (19). Substituting nominal values for these variables leads to a value of β of between 0.5 and 0.8, which could conceivably correspond to the efficiency of a well-designed Brayton engine. The condition that $\bar{m} = 1 - x$ indicates that the ratio of working fluid mass flow in the heat engine to propellant flow is greater than one.

As the value of β increases, it will eventually exceed realistic values for the efficiency of the power cycle. Above values of 80%, it is unlikely that the condition for zero radiator area can be met. Instead, it is desirable to see if any other optimal operating point can be found. With β a prescribed value (as determined previously), there are two free parameters (η, x) that characterize the power system and span the optimization space. Figure 3 shows how the radiator area varies for different values of η and x (T_2/T_3), corresponding to the preceding case where $\beta = 0.7$.

It can be seen that when the efficiency equals the value of β, the area becomes zero for all values of x. For values of η greater than β, the resulting area is negative, which is, of course, not physically meaningful. If β is increased to 0.85, which is assumed higher than can be achieved as an efficiency, Fig. 4 shows that a local minimum does indeed exist at lower values of η. The exact value of the minimum varies slightly as x changes, but is very nearly constant. This allows for a variety of temperature ratios (x) or, equivalently, mass flow ratios ($\bar{m} = 1 - x$) to be established at the same radiator area and at an easily achievable cycle efficiency.

It is, of course, possible that the need for power generation exceeds the value of $\dot{m}_p c_{p_p} T_3$, resulting in β being greater than one. Under

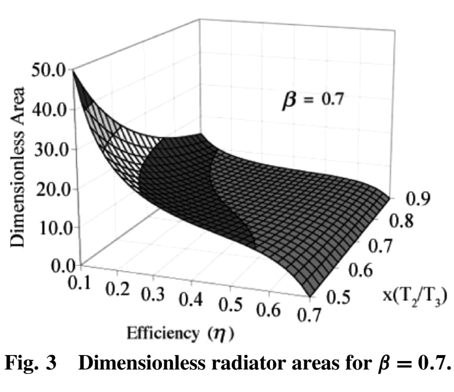

Fig. 3 Dimensionless radiator areas for $\beta = 0.7$.

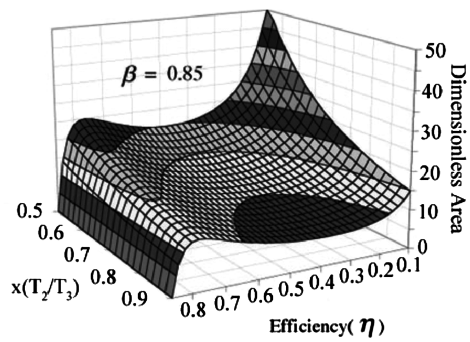

Fig. 4 Dimensionless radiator areas for $\beta = 0.85$.

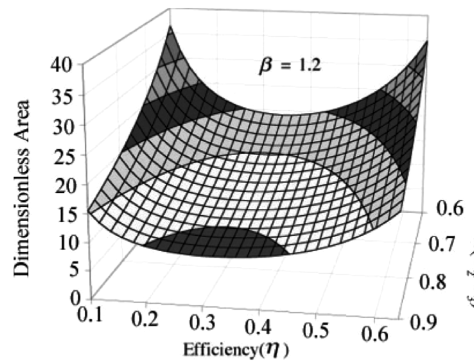

Fig. 5 Dimensionless radiator areas for $\beta = 1.2$.

these conditions, it is again impossible to eliminate the need for radiators to reject heat, however, minimization of their mass can still be achieved. The design space for the case of $\beta = 1.2$ (see Fig. 5) looks very similar to that found in the preceding examples, except that the line where the area goes to zero has moved out of the allowable design space to values of η that are greater than one.

Another possible operating condition would occur at startup, when the power system may be called upon without the assistance of propellant flow to reject waste heat. This condition must be allowed for if the reactor is to be used independently as a power source, apart from a propulsion system. In this mode, the only possible source of heat rejection is by radiators and corresponds to the condition that $\bar{m} = 0$. This last condition requires that a new parameter other than β be used to represent the dimensionless power level (recall that the definition of β had $\bar{m}$ in the denominator). A natural choice is to nondimensionalize the power by $\dot{m}_c c_{p_c} T_3$ in a similar manner to how it was done in the presence of propellant flow. A new parameter β' is introduced and is defined by

$$\beta' = \eta(1-x) = \frac{P_c}{\dot{m}_c c_{p_c} T_3} \tag{21}$$

The expression for the dimensionless radiator area can now be rewritten using this relationship, as

$$\bar{A} = \frac{A}{A_m} = \frac{1}{3\beta'(1-\eta)^3}\left[\left\{1 - \frac{\beta'}{\eta}\right\}^{-3} - 1\right] \tag{22}$$

which can be minimized analytically by differentiating with respect to η while holding β' constant. The result is a cubic equation in β'/η of the form

$$\left(\frac{\beta'}{\eta}\right)^3 - 4\left(\frac{\beta'}{\eta}\right)^2 + \left(6 + \frac{1}{\beta'}\right)\left(\frac{\beta'}{\eta}\right) - 4 = 0 \tag{23}$$

which, given β', can be solved for the cycle efficiency η. Figure 6 shows a plot of $\bar{A}$, η, and x (T_2/T_3) for varying values of β'. Because the mass flow rate of the working fluid found in the denominator of β' is not fixed by other constraints, it is more desirable from the

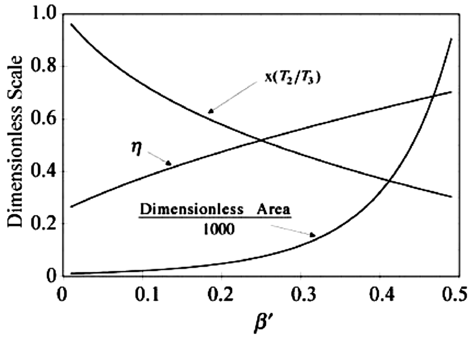

Fig. 6 Dimensionless radiator areas vs β'.

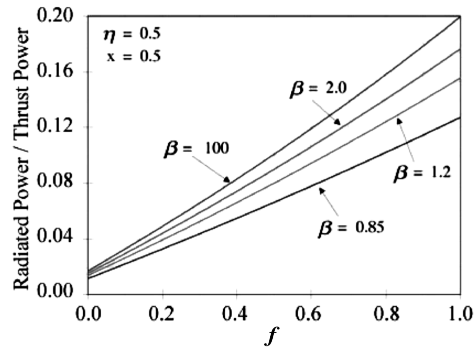

Fig. 7 $P_{\text{rad}}/P_{\text{thrust}}$ vs f.

standpoint of radiator mass to have a high mass flow rate through the heat engine. The efficiency of the heat engine is driven toward lower values, and the ratio T_2/T_3 (also the ratio T_1/T_4) is driven toward higher values. It is the increase in x that offsets the decrease in efficiency, resulting in less waste heat being generated during each cycle.

System Performance

The purely hydrodynamic system discussed earlier had the disadvantage of being limited to a factor of three increase over a solid core system. This was due to requiring that propellant flow be used to regeneratively cool the moderator of the reactor. Addition of power generation provides not only a method of cooling the moderator (decoupling it from the propellant flow), but also a means to put this power back into the flow through the MHD system. There is no coupling in this system's architecture that limits the specific impulse achievable, as there was in the hydrodynamic vortex. The only loss of power to the system is by what is radiated away, because the portion that undergoes heat exchange with the propellant is carried on through to the vortex cavities. In addition to wanting to minimize the radiator mass, it is then also desirable to reduce the fraction of the power that is lost to the radiators.

Two parameters of interest in determining system performance are 1) the ratio of radiated power to thrust power, and 2) the ratio of electric power generated to thrust power. The first is useful in that it can be used to calculate the thrust efficiency of the system. Because under the assumption of complete expansion to vacuum, the only loss of power is through the radiators, the overall system balance can be written as

$$P_{\text{thrust}} = P_{\text{total}} - P_{\text{rad}} \tag{24}$$

and dividing through by the thrust power, the thrust efficiency can then be given by

$$\eta_{\text{thrust}} = \frac{P_{\text{thrust}}}{P_{\text{total}}} = \left[1 + \frac{P_{\text{rad}}}{P_{\text{thrust}}}\right]^{-1} \tag{25}$$

To evaluate this ratio, the thrust power is first written in a second form, based on it being the result of heat added from different sources. If it is assumed that the final thermal state of the propellant corresponds to full expansion to vacuum at a very low temperature, this can be taken as a reference point to the amount of heat that is in the propellant at a given point in the system and the power then evaluated based on its mass flow rate. The propellant therefore starts at a relatively low-energy state in the tank, after which heat is added through counterflow heat exchange with the working fluid, and then by regeneratively cooling the moderator. Because the temperature upon leaving the moderator is known (T_3), the power supplied to the propellant up to this point can be written as $P_{\text{prop}} = \dot{m}_c c_{p_c} T_3$. Power is further supplied by both direct heating from the gas phase fission, and by the power being supplied by the MHD drive. Because the propellant is then exhausted to the zero energy state through the nozzle, the thrust power can be written in the alternative form

$$P_{\text{thrust}} = P_{\text{prop}} + 0.9(1-f)P + P_c \tag{26}$$

Eliminating the total power from the two expressions for thrust power, and dividing through by P_{rad} results in the relationship

$$\frac{P_{\text{rad}}}{P_{\text{thrust}}} = \frac{1 - 0.9(1-f)}{1 + 0.9(1-f) + P_{\text{prop}}/P_{\text{rad}}} \tag{27}$$

The ratio of propellant power (at moderator exit) to radiator power can be written in terms of the optimization variables defined in the last section, to give

$$\frac{P_{\text{prop}}}{P_{\text{rad}}} = \frac{1 - \eta/\beta(1-x)}{(1-\eta/\beta)(1-\eta)(1-x)} \tag{28}$$

For the condition where $\eta = \beta$, this expression goes to infinity, causing the ratio of radiated power to thrust power to go to zero as expected, as the radiator area goes to zero. Choosing various values of β and values of x and η in line with the conditions of optimality found in the last section, Fig. 7 shows how the ratio of radiated power to thrust power varies with the parameter f.

This parameter f, which determines how much fission takes place in the solid region of the reactor, is set by the second parameter of interest mentioned earlier: the ratio of electric power generated to thrust power. This ratio is found in the same manner as the radiated power ratio, but instead, the thrust power is divided through by P_c, and the substitutions for β, x, and η are then made. Only the final equation is presented here and is given by

$$\frac{P_c}{P_{\text{thrust}}} = \frac{1 - 0.9(1-f)}{(1 + 1/\beta) + 0.9(1-f)(1-\eta)(\beta/\eta - 1)} \tag{29}$$

From Eq. (19), a value can be calculated for the ratio of MHD power required to thrust power delivered. Adding all other power requirements, normalized in the same manner, to this ratio will then lead to the ratio of total electric power generated to thrust power. If the target I_{sp} of the system is known, this ratio can be used to determine the value of β through the relationship

$$\beta = \frac{P_c}{\dot{m}_p c_{p_p} T_3} = \left(\frac{P_c}{P_{\text{thrust}}}\right)\frac{(I_{\text{sp}} g)^2}{2 c_{p_p} T_3} \tag{30}$$

This same relationship can also be substituted into Eq. (29) to give an expression for the specific impulse as

$$I_{\text{sp}} = 1/g\{2\beta c_{p_p} T_3/(P_c/P_{\text{thrust}})\}^{1/2} \tag{31}$$

which has been plotted in Fig. 8 against f for several values of β. Given the ratio of P_c/P_{thrust} from Eq. (19) and the specific impulse of the proposed system, this graph allows the determination of the value of f required to supply enough power to the electrical systems. This parameter can, in turn, be used to determine what fraction of the power is being radiated away from the system using Fig. 7. Performance graphs can then be generated for various combinations of optimal η and x values to choose a final system configuration.

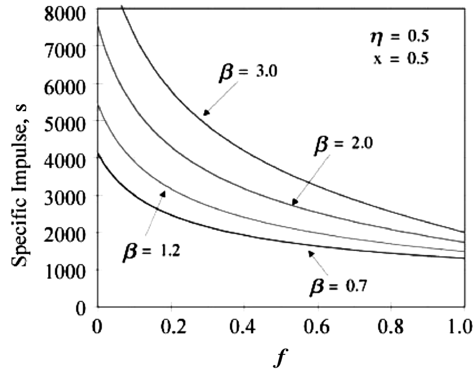

Fig. 8 Specific impulse vs f.

An analysis of this detail for the case when no propellant flows is unnecessary for two reasons. First, no comparison can be made to thrust power, because none is being produced. Second, all the waste heat generated must be radiated away (again because no propellant is available) and is, therefore, simply related to the fission power supplied and the cycle efficiency by

$$P_{\text{rad}} = \frac{1-\eta}{\eta} P_{\text{fission}} \tag{32}$$

Obviously, the higher the cycle efficiency, the less power must be radiated into space. The state of the art and requirements for power while thrusting vs power without thrusting must all come together to determine how the power system and radiators will be sized.

Baseline System Design

Over the course of the analyses leading up to this paper [3,4,7], many parameters have been identified to characterize the system. Some of these have been related to others or constrained to specific values on a physical basis, some were determined through optimization, and some were shown to have little impact on the system performance. In this section, a baseline system will be identified that incorporates these parameters into the current analysis to assess the level of performance that can be achieved by this system. Details of the origin of these parameters and their assumed values can be found in the previous work [3,4,7].

To begin, the propellant pumping pressure will be taken at the state-of-the-art value of 400 atm, which, due to pressure loss through the vortex, will mean a value of pressure at the peak fuel concentration that is about 200 atm. The tangential Mach number at r_m will be taken at a moderate value of 0.7, and w_m will be set at 10. This value will allow for a high uranium loading without excessive pressure drop. From the constraint imposed by the radiative energy transfer [4], this will set the core temperature to just over 5000 K, and the specific impulse to 1200 s.

Taking the optimal value of $\beta_\nu = 0.6$, as well as assuming a right circular cylinder reactor geometry, the thrust and number of vortex tubes is given as 50 kN and 700 tubes,respectively [7]. In addition, previous assumptions yield a mass flow rate per tube of $0.010\ \text{kg/m}\cdot\text{s}$, a tube radius of 2.0 cm, a reactor size of $D = H = 2.0$ m, and a reactor mass of 10 MT. The total mass flow is $4.7\ \text{kg/s}$ and the thrust power of the system is then 340 MW.

Running trials to determine the MHD parameter that will accommodate a stable density profile and zero tangential velocity at the wall, results in MHD = 1.5 and from the optimization on current and electric potential yield $J_p = 22\ \text{A/cm}^2$ and $B = 0.23$ T [4]. The resulting power requirements dictate a β value of 0.7, which is a reasonable number to match the efficiency of the heat cycle, assuming a sufficiently low exit temperature. The exit temperature of the turbine would then be set around 750 K, assuming $T_3 = 2500$ K.

This leads to the question of power system mass. By matching the cycle efficiency to β, the radiator has been eliminated, but no account has been made of the compressor and turbine. To estimate the mass of this system, specific power ratios were considered from existing gas turbine generators. Generally, these systems will range from as low as $1\ \text{kW/kg}$ for tanks, to just over $10\ \text{kW/kg}$ for helicopters. However, recent advances in microturbine power generation predict power densities that may, in theory, reach as high as $1000\ \text{kW/kg}$ [8]. Without relying on these revolutionary increases in performance, one might concede that advances in materials and economies of scale at the higher power levels being considered might allow an increase of 2–4 times over the current state of the art. Assuming the more optimistic end of $40\ \text{kW/kg}$, the power–generation system under consideration would have a mass of about 8.5 MT, which is 85% of the reactor itself. The total mass of the system is then nearly 19 MT, which results in a thrust-to-weight ratio of 27% and a specific thrust power of $18\ \text{kW/kg}$. The baseline system is summarized in Table 2.

Table 2 Baseline system parameters

Parameter	Value
I_{sp}	1200 s
Power	340 MW
Thrust	50 kN
Mass	19 MT
F/W	0.27
Temp	5000 K
Press	400 atm
Diameter	2.0 m
B-field	0.23 T
f	0.15
J_p	$22\ \text{A/cm}^2$
M_{tm}	0.7
r_m	2 cm
Mass flow	$0.01\ \text{kg/m}\cdot\text{s}$
No. of tubes	700
MHD	1.5
w_m	10
β	0.7
β_v	0.6
η	0.7
A_{rad}	0

Conclusions

The last in a series of papers on the effectiveness of using magnetohydrodynamically driven vortices to contain uranium in a gas core nuclear propulsion system has been presented. This paper considered the flow of power throughout the system, from its source in the nuclear fission to its acceleration out of the nozzle. Because part of the fission power must naturally be deposited within the solid walls of the containment structure, the original vortex confinement concept was shown to be limited to 2500 s of specific impulse. The power cycle required by the MHD-driven concept acts to decouple this constraint by drawing heat out of the structure and depositing a major portion of it back into the propellant, however, more stringent constraints on the specific impulse have been identified in a previous paper. It has been shown that under proper design parameters, all of the waste power can be recycled into the propellant and the need for radiators is removed. Although a large amount of recirculated power is required to drive the MHD, the mass of the power system is shown to be comparable to that of the reactor, providing an example system with a thrust-to-weight ratio of 27% and a specific power of $18\ \text{kW/kg}$.

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